TAULA DE FUNCIONS PRIMITIVES

Formes elementals

1. $\displaystyle\int x^n \, dx = \frac{x^{n+1}}{n+1} + C \quad (\text{si } n \neq -1)$
2. $\displaystyle\int (f(x))^n f'(x) \, dx = \frac{(f(x))^{n+1}}{n+1} + C \quad (\text{si } n \neq -1)$
3. $\displaystyle\int \frac{dx}{x} = \ln|x| + C$
4. $\displaystyle\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$
5. $\displaystyle\int e^x \, dx = e^x + C$
6. $\displaystyle\int a^x \, dx = \frac{a^x}{\ln a} + C$
7. $\displaystyle\int \sin x \, dx = -\cos x + C$
8. $\displaystyle\int \cos x \, dx = \sin x + C$
9. $\displaystyle\int \tan x \, dx = -\ln|\cos x| + C$
10. $\displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\frac{x}{a} + C$
11. $\displaystyle\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$
12. $\displaystyle\int \frac{dx}{\sin ax} = \frac{1}{a}\ln\left|\tan a\frac{x}{2}\right| + C$
13. $\displaystyle\int \frac{dx}{\cos ax} = \frac{1}{a}\ln\left|\frac{1}{\cos ax} + \tan ax\right| + C$
14. $\displaystyle\int \frac{dx}{\sin^2 ax} = \frac{-1}{a\tan ax} + C$
15. $\displaystyle\int \frac{dx}{\cos^2 ax} = \frac{1}{a}\tan ax + C$
16. $\displaystyle\int \arcsin ax \, dx = x\arcsin ax + \frac{1}{a}\sqrt{1 - a^2x^2} + C$
17. $\displaystyle\int \arccos ax \, dx = x\arccos ax - \frac{1}{a}\sqrt{1 - a^2x^2} + C$
18. $\displaystyle\int \arctan ax \, dx = x\arctan ax - \frac{1}{2a}\ln(1 + a^2x^2) + C$

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Formes trigonomètriques

19. $\int \sin^2 ax \; dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$

20. $\int \cos^2 ax \; dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$

21. $\int \tan^2 ax \; dx = \frac{1}{a}\tan ax - x + C$

22. $\int \sin ax \cos bx \; dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C \;\; (\text{si } a^2 \neq b^2)$

23. $\int \sin ax \sin bx \; dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C \;\; (\text{si } a^2 \neq b^2)$

24. $\int \cos ax \cos bx \; dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C \;\; (\text{si } a^2 \neq b^2)$

25. $\int \sin ax \cos ax \; dx = -\frac{\cos 2ax}{4a} + C = \frac{\sin^2 ax}{2a} + C' = -\frac{\cos^2 ax}{2a} + C''$

26. $\int \sin^n ax \; dx = \frac{-\sin^{n-1} ax \;\; \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax \; dx$

27. $\int \cos^n ax \; dx = \frac{\cos^{n-1} ax \;\; \sin ax}{na} + \frac{n-1}{n} \int \cos^{n-2} ax \; dx$

28. $\int x \sin ax \; dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax + C$

29. $\int x \cos ax \; dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C$

30. $\int x^n \sin ax \; dx = -\frac{x^n}{a}\cos ax + \frac{n}{a}\int x^{n-1}\cos ax \; dx$

31. $\int x^n \cos ax \; dx = \frac{x^n}{a}\sin ax - \frac{n}{a}\int x^{n-1}\sin ax \; dx$

32. $\int \sin^n ax \cos^m ax \; dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n}\int \sin^{n-2} ax \cos^m ax \; dx$
$(\text{si } m+n \neq 0)$

33. $\int \sin^n ax \cos^m ax \; dx = \frac{\sin^{n+1} ax \cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n}\int \sin^n ax \cos^{m-2} ax \; dx$
$(\text{si } m+n \neq 0)$

34. $\int \tan^n ax \; dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax \; dx \qquad (\text{si } n \neq 1)$

35. $\int \frac{1}{\tan^n ax} \; dx = -\frac{1}{a(n-1)\tan^{n-1} ax} - \int \frac{1}{\tan^{n-2} ax} \; dx \qquad (\text{si } n \neq 1)$

Formes que inclouen exponencials i logaritmes

36. $\int xe^{ax}\ dx = \frac{e^{ax}}{a^2}(ax-1)+C$

37. $\int x^n e^{ax}\ dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}\ dx$

38. $\int e^{ax}\sin bx\ dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)+C$

39. $\int e^{ax}\cos bx\ dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)+C$

40. $\int \ln ax\ dx = x\ln ax - x + C$

41. $\int x^n \ln ax\ dx = \frac{x^{n+1}}{n+1}\ln ax - \frac{x^{n+1}}{(n+1)^2}+C \qquad (\text{si } n \neq -1)$

42. $\int \frac{\ln ax}{x}\ dx = \frac{1}{2}(\ln ax)^2 + C$

Formes hiperbòliques

43. $\int \sinh ax\ dx = \frac{1}{a}\cosh ax + C$

44. $\int \cosh ax\ dx = \frac{1}{a}\sinh ax + C$

45. $\int \tanh ax\ dx = \frac{1}{a}\ln(\cosh ax) + C$

46. $\int \sinh^2 ax\ dx = \frac{\sinh 2ax}{4a} - \frac{x}{2} + C$

47. $\int \cosh^2 ax\ dx = \frac{\sinh 2ax}{4a} + \frac{x}{2} + C$

48. $\int \frac{1}{\sinh ax}\ dx = \frac{1}{a}\ln\left|\tanh\frac{ax}{2}\right| + C$

49. $\int \frac{1}{\cosh ax}\ dx = \frac{1}{a}\arcsin(\tanh ax) + C$

50. $\int \frac{1}{\sinh^2 ax}\ dx = \frac{-1}{\operatorname{arctgh}\ ax} + C$

51. $\int \frac{1}{\cosh^2 ax}\ dx = \frac{1}{a}\tanh ax + C$

Formes que inclouen $\sqrt{x^2 \pm a^2}$

52. $\displaystyle\int \sqrt{x^2 \pm a^2}\, dx = \frac{x}{a}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

53. $\displaystyle\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

54. $\displaystyle\int \frac{\sqrt{x^2 + a^2}}{x}\, dx = \sqrt{x^2 + a^2} - a \ln\left|\frac{a + \sqrt{x^2 + a^2}}{x}\right| + C$

55. $\displaystyle\int \frac{\sqrt{x^2 - a^2}}{x}\, dx = \sqrt{x^2 - a^2} - a \operatorname{arcsec}\left|\frac{x}{a}\right| + C$

56. $\displaystyle\int \frac{dx}{x\sqrt{x^2 + a^2}} = -\frac{1}{a} \ln\left|\frac{a + \sqrt{x^2 + a^2}}{x}\right| + C$

57. $\displaystyle\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arcsec}\left|\frac{x}{a}\right| + C = \frac{1}{a} \arccos\left|\frac{a}{x}\right| + C'$

58. $\displaystyle\int x^2\sqrt{x^2 \pm a^2}\, dx = \frac{x}{8}(2x^2 \pm a^2)\sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

59. $\displaystyle\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\, dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \mp \frac{a^2}{2} \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

60. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2 \pm a^2}} = \mp\frac{\sqrt{x^2 \pm a^2}}{a^2 x} + C$

61. $\displaystyle\int \frac{\sqrt{x^2 \pm a^2}}{x^2}\, dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$

Formes que inclouen $\sqrt{a^2 - x^2}$

62. $\displaystyle\int \sqrt{a^2 - x^2}\, dx = \frac{x}{a}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin\frac{x}{a} + C$

63. $\displaystyle\int x^2\sqrt{a^2 - x^2}\, dx = \frac{a^4}{8} \arcsin\frac{x}{a} - \frac{1}{8}x\sqrt{a^2 - x^2}(a^2 - 2x^2) + C$

64. $\displaystyle\int \frac{\sqrt{a^2 - x^2}}{x}\, dx = \sqrt{a^2 - x^2} - a \ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right| + C$

65. $\displaystyle\int \frac{\sqrt{a^2 - x^2}}{x^2}\, dx = -\frac{\sqrt{a^2 - x^2}}{x} - \arcsin\frac{x}{a} + C$

66. $\displaystyle\int \frac{x^2}{\sqrt{a^2 - x^2}}\, dx = \frac{a^2}{2} \arcsin\frac{x}{a} - \frac{1}{2}x\sqrt{a^2 - x^2} + C$

67. $\displaystyle\int \frac{1}{x\sqrt{a^2 - x^2}}\, dx = -\frac{1}{a} \ln\left|\frac{a + \sqrt{a^2 - x^2}}{x}\right| + C$

68. $\displaystyle\int \frac{1}{x^2\sqrt{a^2 - x^2}}\, dx = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$

Formes que inclouen $ax+b$ o bé $\sqrt{ax+b}$

69. $\displaystyle\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a^2}\left(\frac{ax+b}{n+2} - \frac{b}{n+1}\right) + C \qquad (\text{si } n \neq -1, -2)$

70. $\displaystyle\int \frac{x}{ax+b}\,dx = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b| + C$

71. $\displaystyle\int \frac{x}{(ax+b)^2}\,dx = \frac{1}{a^2}\left(\ln|ax+b| + \frac{b}{ax+b}\right) + C$

72. $\displaystyle\int (\sqrt{ax+b})^n\,dx = \frac{2}{a}\frac{(\sqrt{ax+b})^{n+2}}{n+2} + C \qquad (\text{si } n \neq -2)$

73. $\displaystyle\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{1}{x\sqrt{ax+b}}\,dx$

74. $\displaystyle\int \frac{1}{x\sqrt{ax+b}}\,dx = \frac{2}{\sqrt{-b}}\arctan\sqrt{\frac{ax+b}{-b}} + C \qquad (\text{si } b < 0)$

75. $\displaystyle\int \frac{1}{x\sqrt{ax+b}}\,dx = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C \qquad (\text{si } b > 0)$